Universal enthalpy-entropy compensation rule for the deformation of metallic glasses

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The thermodynamic compensation law describing an empirical linear relationship between activation enthalpy and activation entropy has seldom been validated for amorphous solids. Here molecular dynamics simulations reveal a well-defined enthalpy-entropy compensation rule in a metallic glass (MG) over a wide temperature and stress range, spanning the glass transition induced by temperature and/or stress. Experiments on other MGs reproduce this law, suggesting that it applies universally to amorphous solids, so we extend it from crystals to amorphous solids. In the glassy state, the compensation temperature is found to agree with the thermal glass transition temperature \( T_g \); whereas in the supercooled liquid region, the compensation temperature matches \( \sim 1.4 T_g \), at which the diffusion kinetics start to feel the roughness of the free-energy surface.

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I. INTRODUCTION

The ubiquitous thermodynamic enthalpy-entropy compensation rule was initially discovered in chemistry [1,2] and biology [3], and was then extended to condensed-matter physics [4] and materials science [5–8]. It indicates a linear relationship between activation enthalpy and activation entropy [9–11]. Despite decades of studies of this effect in crystals, it has not been validated in amorphous solids. In this paper, combined atomistic simulations and experiments reveal a universally well-established enthalpy-entropy compensation in the deformation of metallic glasses (MGs), although the structure and deformation units of these two phases differ sharply from each other [12–14].

Transition state theory (TST) is a general statistical law explaining the rates of chemical reactions [15]. It is nowadays prevalent to describe the plastic deformation of material, which is usually driven by thermally activated processes [16,17]. TST predicts the occurrence frequency of a plastic event using the Arrhenius equation, which associates the activation Gibbs free energy \( \Delta G \) with the resultant rate \( v \), i.e.,

\[
v = v_0 \exp \left( \frac{-\Delta G(T)}{k_B T} \right). \tag{1}
\]

For shear-stress-driven deformation processes, the rate is a function of the shear stress \( \tau \) and temperature \( T \). \( v_0 \) is an attempt frequency. Further, \( k_B \) is Boltzmann’s constant and \( k_B T \) is the thermal energy. Finally, \( \Delta G \) is

\[
\Delta G(T) = \Delta Q - \tau \Omega - T \Delta S. \tag{2}
\]

Here, \( \Delta Q \) is the activation energy and \( \Omega \) is the activation volume. \( \Delta H = \Delta Q - \tau \Omega \) denotes activation enthalpy and \( \Delta S \) is the activation entropy. The above theoretical framework constitutes the fundamentals of material deformation physics.

The thermodynamic compensation rule, or Meyer-Neldel (MN) rule [9], indicates an empirical linear relationship between the activation entropy and activation enthalpy,

\[
\Delta S = \frac{\Delta H}{T_{MN}}, \tag{3}
\]

where \( T_{MN} \) is a characteristic temperature called the MN temperature [10,11]. MN rule predicts \( \Delta G = (1 - T/T_{MN}) \Delta H \). It has been found to be ubiquitous in a vast number of chemical phenomena [18], condensed-matter physics [4], and biological processes [3]. Recently, it has been more frequently reported in the plastic deformation mechanisms of crystals, e.g., diffusion [8], interface and grain boundary mobilities [5], and dislocation behaviors [6,7].

Although the MN rule seems to apply universally to crystals, it is interesting to know whether it stands or breaks down in amorphous solids [19]. To date, a compensation effect has been reported for diffusion in MGs between the diffusion prefactor and activation enthalpy [20]. In contrast with crystals, the deformation unit of MGs has not been completely recognized [21]. At low temperature, the plastic event is commonly assumed to be a collective local shear of atoms. The scenarios include the shear transformation zone (STZ) [22,23], cooperative shear model (CSM) [24,25], flow unit [26–29], and tensile transformation zone (TTZ) [30,31]. The STZ represents a thermally activated displacive process [32,33] with a typical activation energy of multiple tens of \( k_B T \) and an activation volume of dozens of atoms [34–36]. Further, at high temperature, a diffuse arrangement of atoms is also involved. In this work, we confirm the validation of the enthalpy-entropy compensation effect in MGs over a wide temperature and stress range, and discuss the possible correlation of the compensation effect with important critical temperatures in MGs.

II. METHODOLOGY

A. Atomistic simulations

The present MD simulation is performed using the LAMMPS code [37]. An embedded-atom method (EAM) potential, which has been extensively verified by both first-principles
FIG. 1. (Color online) Atomic shear deformation of Cu50Zr50. (a) Schematic sketch of the atomistic shear deformation. (b) MD snapshots indicating homogeneous shear deformation with strain rate $10^8 \text{ s}^{-1}$ and temperature 600 K. Atoms are colored according to their local von Mises strain $\eta_i$ of atom $i$ [47], which is implemented in ATOMEYE [48]. Typical stress-strain curves at (c) 550 and (d) 750 K, respectively.

calculations and experiments [38], is adopted to describe the interatomic interaction. This potential has been widely used to investigate the structure and mechanics of many multicomponent MGs [39–41]. Here a Cu50Zr50 glass model is prepared by a melting-quenching technique with a cooling rate of $10^{12} \text{ K/s}$. The model contains 100,000 atoms, with dimensions of $20 \times 20 \times 4 \text{ nm}^3$ for the $X$, $Y$, and $Z$ directions, respectively. The glass transition temperature of Cu50Zr50 is estimated as $T_g = 675 \text{ K}$ by MD, which is in agreement with experimental data [42,43]. Note that in MGs, shear is the dominant deformation mode, even though compression or tension is more frequently applied in experiments [22,34,44]. Therefore, we conduct straightforward shear deformation in these computer simulations. Shear deformation is applied with a constant strain rate between $10^7$ and $10^{10} \text{ s}^{-1}$ for the engineering shear strain $\gamma_{xy}$; see Fig. 1(a). The constant strain rate is applied by incrementally shearing the simulation box at each MD step. The MD time step is set to be 2 fs. All of the other stress components except $\tau_{xy} = \tau_{yx}$ are kept at 0 during deformation. Simulations are conducted within an isothermal-isobaric ensemble via a Parrinello-Rahman technique [45]. All of the samples are thermally equilibrated sufficiently for 200 ps before shearing. We choose the temperature range of [500 K, 850 K], which is around the $T_g$ value of Cu50Zr50. The above conditions guarantee that the deformation lies on the homogeneous flow regime of the MG deformation map [34], where thermal activation is believed to dominate the plastic deformation [46].

B. Activation free energy

In a steady-state flow, the effective shear rate $\dot{\gamma}$ usually obeys a general rate law [22,34,49]:

$$\dot{\gamma} = \dot{\gamma}_0 \exp \left[ -\frac{\Delta G(\tau,T)}{k_B T} \right].$$  \hspace{1cm} (4)

For the STZ, $\dot{\gamma}_0 = a_0 v_0 \gamma_0$ is a shear rate prefactor that establishes the upper limit of the strain rate. $a_0$ (of order unity) is a parameter incorporating the volume fraction of glass that is ready to deform and other numerical factors [22,34]. $v_0 \sim 10^{11} - 10^{13} \text{ s}^{-1}$ is the attempt frequency, which depends on the diameter of the deformation unit [49]. $\gamma_0 \sim 0.1$ is the unit strain at which STZs transform on average [22,34,46]. The above considerations yield a reasonable characteristic shear rate on the order of $10^{10} - 10^{12} \text{ s}^{-1}$, which is commonly assumed in the literature [22,34,50]. Our MD predicts $\dot{\gamma}_0 = 1.2 \times 10^{10} \text{ s}^{-1}$ (see the details in Fig. S1 of the Supplemental Material [51]), which agrees with the above analysis [22,34]. If we rewrite Eq. (4) as

$$\Delta G(\tau,T) = k_B T \ln (\dot{\gamma}_0'/\dot{\gamma}),$$  \hspace{1cm} (5)

then $\Delta G$ can be derived as a function of both $T$ and $\tau$ on the basis of the plots in Fig. 2(a). Note that a change of even one order of magnitude in $\gamma_0$ will cause variations of only $\sim 2k_B T$ in the free energy and $\sim 2k_B T$ in the entropy; the uncertainty in the prefactor $\dot{\gamma}_0$ does not substantially change the main conclusions based on the free energy, which is multiple tens...
of $k_B T$. By employing Eq. (4), we have taken the statistics of competing processes with scattered energy barriers as a single activation process in MG deformation, which has been theoretically verified previously by an activation-relaxation technique in MD [19].

III. RESULTS

A. Temperature- and stress-driven transition of deformation mechanisms

We simulate Cu$_{50}$Zr$_{50}$ MG shear deformation over a wide temperature range (500–850 K) that extends both below and above $T_g = 675$ K, which is the stress-dependent glass transition temperature [39] at the stress free condition: $T_g(\tau)|_{\tau=0} = T_g(0) = T_g$. This temperature range allows for the homogeneous shear of the glass model; see Figs. 1(a) and 1(b) for simulations [51]. Figures 1(c) and 1(d) show that the flow stress is more strain-rate sensitive at high temperature, which indicates that the deformation unit becomes less collective (activation volume is inversely proportional to the rate sensitivity).

The strain rate is displayed as a function of the flow stress at different temperatures in Fig. 2(a). The inset shows the variation in the stress exponent $\gamma$ that is theoretically verified previously by an activation-relaxation technique in MD [19].

The inset of Fig. 2(a) shows that Newtonian flow could be obtained only in the low-strain-rate and high-temperature regime, which is marked by a stress exponent $n = 1$ (if a power law of $\dot{\gamma} \propto \tau^n$ is assumed, and $n = \partial \ln \dot{\gamma} / \partial \ln \tau$). However, the majority of the deformation shows a collective nature, indicated by a large stress exponent ($n > 1$, non-Newtonian flow). The inset of Fig. 2(a) also shows that the stress exponents are different at high- and low-strain-rate regimes, such as $\dot{\gamma} > 5 \times 10^8$ s$^{-1}$ and $\dot{\gamma} < 5 \times 10^8$ s$^{-1}$. The threshold $\dot{\gamma} \sim 5 \times 10^8$ s$^{-1}$ is empirically chosen from the kink of the rate-stress plot.

The transition of the stress exponent indicates the temperature- and stress-induced transition of deformation mechanisms. Figure 2(b) shows that the activation volume $\Omega = -\partial \Delta G / \partial \tau$, based on Figs. 3(a) and 3(b) below, is dramatically decreased by increasing the temperature above $T_g(0)$; thus, the thermal glass transition actually introduces the transition in the deformation mechanism. The size of the activation volume here is similar to that estimated experimentally [35]. At high temperature, viscous flow involving a diffusive mechanism dominates the deformation. Figure 2(c) shows that stress can also induce the transition in the deformation mechanism. The activation volume at 500 K (derived as $\Omega = k_B T \partial \ln \dot{\gamma} / \partial \tau$) varies from about 500 Å$^3$ to about 100 Å$^3$ with increasing stress. The latter resembles the activation volume after the thermal glass transition, as shown in Fig. 2(b), which suggests that stress could also drive a glass transition even below $T_g(0)$. This scenario of a stress-induced glass transition agrees with early speculation [52] and atomistic demonstration [39]. Figure S2 of the Supplemental Material provides further evidence for this point [51].

The viscosity contour $\eta(\tau, T) = \tau/\dot{\gamma}_0 \exp(\Delta G/k_B T)$ shown in Fig. 2(d) intuitively characterizes the glass transition induced by temperature, stress, or both. Temperature and stress

FIG. 2. (Color online) Temperature- and stress-driven transition of deformation mechanisms in MG. (a) Correlation between strain rate and flow stress. Inset shows the variation in the stress exponent $\gamma$. (b) Activation volume decreases abruptly at temperature well below $T_g(0)$, denoting a temperature- and stress-induced glass transition. (c) Logarithmic strain rate vs stress. Slope change indicates a decrease in activation volume, which implies a stress-induced glass transition. (d) Viscosity contour against both temperature- and stress-induced glass transition and a shift in the deformation mechanism.
The shaded area in Fig. 3(a) implies a transition in mechanism. Inset shows that the activation volume is in agreement temperature. Data in the shaded area are excluded due to change agreement.

Finally, we can confirm the enthalpy-entropy correlation in MGs. Figures 3(c) and 3(d) present well-defined linear relationships between $\Delta S$ and $\Delta H$ for the two regimes. The characteristic temperatures $T_{MN}$ are determined from linear fittings as (I) $T_{MN}^{\text{I}} = 687 \pm 51$ K and (II) $T_{MN}^{\text{II}} = 970 \pm 40$ K, respectively. In the glassy state, $T_{MN}^{\text{II}} = 687 \pm 51$ K for the dominant mechanism below $\tau_c(T)$. It is very close to $T_e(0) = 675$ K. In the supercooled liquid regime, the compensation temperature is about $1.4T_e(0)$.

C. Experimental support for enthalpy-entropy compensation rule in MGs

The MD informed compensation rule is also supported by compressive experiments on Vitreloy 1 (Vit 1) by Lu et al. [54], as shown in Fig. 4. Figure 4(a) shows how the flow stress is adjusted by the strain rate. Since compression is implied in experiments, we redefine $\Delta G = \Delta H - T \Delta S = \Delta Q - \sigma\Omega - T \Delta S$, where $\sigma$ is the compressive stress and $\Omega$ is its conjugate activation volume. $\Delta G$ could be obtained before and after $\tau_c(T)$. On the other hand, Fig. 3(b) shows that $\Delta G$ versus $\tau$ could be described well by a single curve when $T > T_e(0)$, indicating no change in the mechanism. As a result, the available $\Delta G(T, \tau)$ has been divided into two regimes, including the data before and after the glass transition, as proposed in Fig. 2(e), regardless of whether the transition is driven by temperature or stress. Then we extract $\Delta G$ as a function of $T$ for multiple stress levels; see Fig. S3 in Supplemental Material [51]. Linear fits based on Eq. (2) yield $\Delta S$ and $\Delta H$.

B. Enthalpy-entropy compensation discovered in MG through atomistic simulations

Next, we demonstrate the enthalpy-entropy compensation for each regime. Figures 3(a) and 3(b) plot $\Delta G$ obtained from Eq. (5) against the stress at temperatures below and above $T_e(0)$, respectively. A frequently used empirical power-law equation, $\Delta G = \Delta F[1 - (\tau/\tau_e(0))^p]^q$, is employed to interpolate the free-energy data points [24,49,53]. Here, $\Delta F = \Delta Q - T \Delta S$ is the activation Helmholtz free energy, $\tau_e(0)$ is the athermal stress, and $0 < p \leq 1$ and $q \geq 1$ are parameters. The shaded area in Fig. 3(a) implies a transition in mechanism induced by stress. For an accurate description, we use separate curves with different $p$ and $q$ to fit the data in Fig. 3(a).
using \( \Delta G(\sigma, T) = k_B T \ln (\dot{\varepsilon}_0/\dot{\varepsilon}) \), where \( \dot{\varepsilon} \) is the compressive strain rate. Here we take the reference strain rate \( \dot{\varepsilon}_0 \) as half of \( \gamma_0 \) because of a geometrical consideration \( [22, 24, 34] \). \( \Delta G \) is plotted against the stress at different temperatures in Fig. 4(b). For a first approximation, we extract \( \delta \) and \( \Delta h \) from the experimental data. The \( \Delta G \) data are further shown versus temperature at different stress levels in Fig. 4(c). Linear fits of \( \Delta G \) versus \( T \) yield the \( \Delta S \) and \( \Delta H \) values. The data in the shaded area below \( T_\text{c}(0) = 624 \text{ K} \) are excluded because the mechanism changes near \( T_\text{c}(0) \), so that all of the experimental data are within regime II, as shown in Fig. 2(e). This is supported by the activation volume shown in the inset of Fig. 4(c). The derived activation volume of the order \( 151 \pm 25 \text{ Å}^3 \) is in nice agreement with the theoretical prediction of the order of \( 100 \text{ Å}^3 \) in regime II; see Fig. 2(b).

Figure 4(d) plots all of the summarized data for the activation entropy versus the activation enthalpy of Vit I \([54]\). A linear relationship seems to be established between \( \Delta S \) and \( \Delta H \). To verify the findings in simulations, we provide two linear plots with a best fit of the compensation rule using the following two linear plots with a best fit of the compensation rule using the compensation temperature \( T_{\text{c} \text{MN}} = T_\text{c}(0) = 624 \text{ K} \) for regime I and \( T_{\text{c} \text{MN}} = 1.4 T_\text{c}(0) = 874 \text{ K} \) for regime II, respectively. While both of the theoretical predictions are within the error range, \( T_{\text{c} \text{MN}} \) is in a better agreement with the experiment than \( T_\text{c} \). The experimental data are basically located in regime II considering the relatively high stress level \( \sigma/E = 0 \sim 0.016 \), where \( E = 96 \text{ GPa} \) is the Young’s modulus of Vit I \([54]\) and the temperature range is above \( T_\text{c}(0) \). The quantity of activation volume shown in the inset of Fig. 4(c) also supports the point that these experimental data are within the supercooled liquid region.

The above analysis confirms the compensation rule using both atomistic simulations and experiments. The very high time resolution of MD allows us to study thermally activated processes happening during even an ephemeral homogenous deformation of an MG, which cannot be captured in experiments. However, MD can be used only for very short times and fast events. On the other hand, experiments enable the study of long time periods and slow events. However, the heterogeneous strain-rate distribution of localized shear deformation following homogenous deformation hinders a purely thermodynamic understanding of the intrinsic deformation unit of MGs based only on experiments. Hence, a study combining MD simulations and experiments provides an enthalpy-entropy compensation effect over a wide strain-rate regime. We also note that strain rate can sometimes change the deformation mechanisms of both crystals \([53, 55-57]\) and amorphous solids \([58]\), which hinders a direct interpretation of experimental phenomena by atomistic simulations due to the time-scale limitation of the latter. The present study compares the high-strain-rate simulations with high-temperature experimental results, which is meaningful in terms of intrinsic homogenous flow behavior of MGs which are accommodated by STZs or other diffusive mechanisms.

### IV. DISCUSSION

Before we discuss any physical meaning of the compensation rule and compensation temperatures in MG, we carry out statistical analysis which indicates a physical cause behind the identified compensation effect in MGs; see details in Fig. S4 of the Supplemental Material \([51]\). We may understand the compensation rule in terms of basic thermodynamics. By combining Eqs. (2) and (5), the activation enthalpy can be expressed as \( \Delta H = T \Delta S + k_B T \ln (\dot{\gamma}/\dot{\gamma}) \). When the temperature approaches \( T_{\text{MN}} \), the strain rate \( \dot{\gamma} \) will reach its upper limit, i.e., \( \gamma_0 \). If the activation entropy is assumed to be insensitive to temperature [as shown in Fig. S3 of the Supplemental Material \([51]\) and Fig. 4(c)], the above expression yields the original form of the MN rule, i.e., \( \Delta S = \Delta H/T_{\text{MN}} \). The physical meaning seems to be that \( \Delta G \) of a specific deformation mechanism tends to be a negligible value when the temperature approaches \( T_{\text{MN}} \), and the deformation mechanism is dominant and holds up to \( T_{\text{MN}} \). However, the deformation mechanism may already shift before reaching \( T_{\text{MN}} \). Therefore, \( T_{\text{MN}} \) can be a theoretical upper limit that specifies a unique deformation mechanism in a specific regime of the deformation map.

To further understand the physical meaning, we analyze the temperature-dependent diffusion kinetics. Figure 5 shows the Arrhenius plot of both Cu and Zr atoms against the reciprocal temperature. At the high-temperature limit, diffusion obeys an Arrhenius relation in the liquid region. At a critical temperature \( T_{\text{c}} \) \( \sim 950 \text{ K} \), a kink appears in the Arrhenius plot. Diffusion starts to deviate from the linear relationship once it feels the roughness of the free-energy surface, while the inset of Fig. 5 shows the cooling history, which yields \( T_{\text{c}}(0) = 675 \text{ K} \), coinciding with \( T_{\text{c} \text{MN}} \).

Therefore, \( T_{\text{c} \text{MN}} \) is in agreement with the thermal glass transition temperature. This is not surprising since the glass transition occurs at \( T_\text{c}(0) \), where structural changes occur and the deformation mechanism shifts. \( T_{\text{c} \text{MN}} = 970 \pm 40 \text{ K} \) matches the critical temperature \( T_{\text{c}} \) \( \sim 950 \text{ K} \) \( \sim 1.4 T_\text{c}(0) \),
where diffusion deviates from the conventional Arrhenius relation. The deviation indicates that diffusion kinetics start to feel the roughness of the free-energy landscape, which also suggests a deformation mechanism shift at this critical temperature. In this sense, $1.4T_g(0)$ is the critical temperature, above which the supercooled liquid loses the memory of previous thermal and mechanical histories [59]. Therefore, $T_{MN}^{II}$ could be a unique theoretical temperature indicative of the deformation mechanism in the supercooled liquid state (regime II). For comparison, we also conduct mode-coupling theory (MCT) analyses in Fig. S5 of the Supplemental Material [51,60,61]. The MCT critical temperature is $T_c \sim 780$ K for Cu$_{50}$Zr$_{50}$, which is well below the estimated $T_{MN}^{II}$ but above $T_{MN}^{IV}$.

V. CONCLUSIONS

To summarize, atomistic simulations and experiments uncover a universal enthalpy-entropy-compensation rule in MGs. It stands in a wide temperature and stress range spanning the glass transition induced by temperature and/or stress. Distinct compensation temperatures are discovered for each regime of the deformation map. Before the glass transition, $T_{MN}$ is correlated with $T_g(0)$, which could be a theoretical upper temperature limit of the STZ. On the other hand, $T_{MN} \sim 1.4T_g(0)$ for supercooled liquid is possibly the theoretical upper temperature limit of the $\alpha$ relaxation, where the diffusion deviates from the Arrhenius relationship because the free-energy surface becomes rough.

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